Numerical simulation of wave propagation in soil media

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ABSTRACT

The propagation of waves in soil media is an important phenomenon in the field of dynamics of soils. Continuum models represent the soil deformation by field equations. In order to investigate the dynamic behavior of soil media, one-dimensional wave propagation is a common benchmark case, which has been analytically solved by other authors and is used as a verification example for linear material model of the proposed approach. In order to simulate the transmitting boundaries absorbing infinite elements are developed and implemented in the numerical analysis. The numerical modeling of the problem is performed by the general finite element software ANSYS, using the user programmable features (UPF) and developed modules for modeling of soil behavior. In order to reflect the propagation of waves numerical analysis is done by extensive parametric study using the infinite elements. The obtained results emphasise some key issues regarding the soil deformation during the wave propagation.

Keywords: numerical analysis, wave propagation, validation

1 INTRODUCTION

The wave propagation phenomenon is an important subject in the field of geotechnical engineering where soil media have to be treated as continua. Dynamic disturbances are best described in the viewpoint of wave propagation. Experiences in numerical simulation of wave propagation show that one of the most difficult parts of the problem is the simulation of boundaries. This is the main point that makes soil dynamics more difficult than structural dynamics since the waves at the boundary must radiate and not return to the medium of interest. Efficient modeling of infinite media is important for many engineering problems. In this work the coupled computational method of finite and infinite elements is used to simulate the wave propagation problem. Implementation is done in the case of one and two dimensional wave propagation. The calculations in this field are performed in time domain. In the implementation of numerical calculations the general finite element software ANSYS [1] is used, applying its User Programmable Features (UPF) programming new elements is possible. Using the UPF absorbent infinite elements have been programmed and implemented in the numerical calculations. In order to simulate the wave propagation numerically the domain of propagation is considered as a combination of finite elements of ANSYS and newly programmed absorbent infinite elements to represent the exterior unbounded region. In the next sections the wave propagation is studied in

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simple one dimensional then extended to two
dimensional cases. The newly programmed
absorbing infinite elements show good
correspondence with finite elements of the
software ANSYS in the sense of continuity of
displacement field at the interface.

2 ABSORBING INFINITE ELEMENTS

Infinite and finite elements can be used together
to model soil media in an efficient way. The
formulation of infinite elements is the same as
for the finite elements in addition to the mapping
of the domain. Infinite elements were first
developed by Zienkiewicz et al. [2] and since
then they have been developed in both frequency
and time domain. Absorptive infinite elements
have been proposed by Häggblad et al. [3] that
can be used in time domain. In this work the
development of infinite element has followed a
similar techniques as in [3] where the infinite
element is obtained from a six noded finite
element as shown in Figure 1.

For coordinate interpolation in \( r-s \) coordinate
system a one-dimensional mapping is applied.

\[
\begin{align*}
M_1 &= - \frac{(1-s)r}{1-r} \\
M_2 &= - \frac{1}{2} \frac{(1-s)(1+r)}{1-r} \\
M_3 &= - \frac{(1+s)r}{1-r} \\
M_4 &= - \frac{1}{2} \frac{(1+s)(1+r)}{1-r}
\end{align*}
\]  (4)

In expression (3) \( r \) and \( s \) are vectors of nodal
point displacements in local coordinates where it
is to be mentioned that on the side of infinity
\((r=1)\) no mappings have been assigned to the
nodes as it is assumed that no displacement is
possible at infinity. Construction of element
matrices is done by using the standard
procedures as described in the work of Bathe [4].

The new coordinate interpolation functions
are taken into consideration in the Jacobian
matrix (Bettess [5]). The approximation for the
element integrals is done by Gauss quadrature
formulas.

For the absorbing layer of the infinite element
the Lysmer-Kuhlmeyer approach [6] is used. In
all cases a plane strain two dimensional situation
is studied. For the impact of plane waves on
element sides normal and tangential stresses are
derived as:
where \( c_p \) and \( c_s \) indicate compression and shear waves, \( \rho \) is the density of soil medium. In order to take into account the directions of the incident waves coefficients \( a \) and \( b \) as suggested by White et al. [7] are used as multipliers for better numerical results. Transformation from local to global coordinates is done automatically by the software ANSYS such that there is no need of defining transformation matrices.

By bringing together the contributions from each element the governing incremental equations for equilibrium in dynamic analysis are obtained. Time derivatives are approximated by Newmark’s method and equilibrium iterations are used in each step. In the following headings one and two dimensional wave propagation problems are analyzed.

3 ONE DIMENSIONAL WAVE PROPAGATION

In order to verify the absorbing properties of the newly programmed infinite elements one dimensional wave propagation is performed using the soil properties as given in Plaxis [8]. The soil column is simulated in two alternative ways. First only finite elements with fixed boundaries and then the same finite elements with infinite element boundaries are considered. In Figure 2 soil domain with point A in the middle is presented. The soil domain of interest has a length of 10 m and is discretized with 40 elements. The properties of soil are given in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Soil properties</th>
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<tbody>
<tr>
<td>Young’s modulus ( E )</td>
</tr>
<tr>
<td>Poisson’s ratio ( \nu )</td>
</tr>
<tr>
<td>Density ( \rho )</td>
</tr>
</tbody>
</table>

As given in [8] the P wave velocity is:

\[
V_p = \sqrt{\frac{(1 - \nu)E}{\rho \left((1 + \nu)(1 - 2\nu)\right)}}
\]

Using the values from Table 1 \( 99V_p = 99 \) m/s is obtained. In order to give a more general picture of 1D wave propagation vertical displacement on top of the domain is applied as three different types of functions:

- Heaviside step function
- Impulse function
- Sine function

3.1 Heaviside step function

The first type of displacement application is considered to be a Heaviside step type where displacement of \( u_y = 0.001 \) m at the very beginning of the time is applied. In Figure 3 the time history of the traveling P wave in the middle point (point A) is given.

As can be seen from Figure 3 in the case of finite elements the wave is reflected while using the infinite elements the wave is absorbed in the boundary so that the value of vertical displacement oscillates around 0.001 m. It is to be mentioned that the time needed for the P-wave (\( V_p = 99 \) m/s) to reach the point A is 0.05 s which is observed in the Figure 3 revealing the correctness of numerical simulation.
3.2 Impulse function

In the following case, application of displacement is considered as an impulse function where application is done only for 0.00166 s and then the displacement is removed. The system is analyzed for 0.3 s in order to see the time-displacement behavior at the middle point of the domain. In Figure 4 the obtained displacement time history at the middle point (Point A) is given:

From the figure it is clearly seen that in the case of finite elements only the impulse is reflected at 0.15 s while using infinite elements the reflection of the wave is reduced considerably.

3.3 Sine function

In order to simulate the displacement as sine function the amplitude of the sine function is taken as 0.001 m with a circular frequency of 0.5 Hz. As in the previous cases displacement is applied at the top of the model. The results in Figure 5 show that the sine wave in case of finite elements only is reduced twice due to the boundary conditions at the bottom of the domain. However, in the case of infinite elements the sine wave has obtained its original magnitude meaning that the infinite elements as boundary did not impact the sine wave amplitude.

4 TWO DIMENSIONAL WAVE PROPAGATION

In case of two dimensional wave propagation a quarter-space is taken into consideration. The material parameters are taken the same as in Table 1. Boundary conditions limit displacements in the free field such that in the soil medium no surface wave propagation is allowed. The soil medium is presented as a combination of finite and infinite elements and is shown in Figure 6. The prescribed displacement at the upper top part of impulse type as in case of 3.2 is considered.
In Figure 7 propagation of waves is shown where it is observed that in case of infinite elements the reflections are much smaller.

5 RAYLEIGH WAVE PROPAGATION

In order to simulate surface wave propagation in 2D Rayleigh wave propagation is considered. In this problem the soil medium composed of finite and infinite elements as shown in Figure 8. In Table 2 soil parameters are given from which P wave velocity is calculated as $V_p = 34.6 \text{ m/s}$.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E$</td>
<td>1000</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$v$</td>
<td>0.25</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>2.0</td>
</tr>
</tbody>
</table>

It is important to note that the soil body is surrounded by infinite elements. Two points namely A(4.03,0) and B(6.08,0) are taken into
consideration. In Figure 9 displacement-time histories for both points are given.

As it is seen from Figure 9 the time difference between points A and B is 0.109 s while the distance between them is 2.05 m as a result of the meshing of the domain. It follows that the velocity of Rayleigh wave is \( V_R = \frac{2.05}{0.109} = 18.8 \) m/s.

The velocity of Rayleigh wave is given as \( V_R = 0.54 V_p = 0.54 \times 34.6 = 18.7 \) m/s in [8] which is almost as the Rayleigh wave velocity obtained from numerical analysis in ANSYS. Thus it can be stated that numerical analysis of Rayleigh wave propagation using finite and infinite elements are performed in a correct way.

6 CONCLUSION

In this work the coupled computational method of finite and infinite elements has been presented. For wave propagation problems the local region of interest is modeled by finite elements, which enable simulation of more complex geometries. On the other hand the surrounding filed of the domain is considered using the absorbent infinite elements so that no reflection of the waves in numerical simulations occurs. These elements have the capability to simulate the infinite region very well. In case of 1D and 2D wave propagation the obtained results show that the usage of absorbent infinite elements improves the results significantly. In numerical simulations ANSYS software is applied. Using its programmable features programming new elements such as the absorbing infinite elements is possible. The obtained numerical results are reliable and further application of coupled finite and infinite elements can be considered in the field of soil structure interaction. Since the programmed absorbent infinite elements are defined in time domain, non-linearity of materials can also be simulated in the finite element region.

REFERENCES